

# Multivariable predictive controller for a test stand of air conditioning<sup>1</sup>

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## 1. Introduction

The technological development of modern aircrafts leads to more and more sophisticated air conditioning equipments : for instance, the new fighter aircrafts will have a specific calculator to regulate the cockpit temperature and pressure. Therefore it is necessary to develop more and more advanced test stands in order to simulate at ground level the running conditions of the equipments.

An industrial system called P2T2 has been perfected to guarantee the certification of the various aircraft equipments. It can generate an air flow at given temperature and pressure controlling two high-pressure air sources by two valves. The regulator which is developed at the present time uses two PID (Proportional Integral Differential) controllers : one acts on pressure deviation and the other on temperature deviation. Yet, the results are not satisfactory owing to important coupling effects and insufficient dynamic performances (see tests on figure 5).

The predictive control, introduced in the eighties, is now soaring in the industrial environment. Many successful applications have been reported in literature : for instance by Clarke (1988), Soeterboek (1992) and Richalet (1993). Numerous theoretical works have also been conducted at ENSICA regarding advanced predictive controllers (Aymes *et al.* 1996). Taking into account all these works, this article proposes a multivariable predictive control (MPC) able to answer the problems raised by this industrial test stand.

The content is organized as follows. Section 2 describes the industrial process. Section 3 presents the MPC algorithm. A multivariable identification of the test stand is developed in section 4. And finally, comparisons between the MPC and the existing PID regulation are reported in section 5.

## 2. The industrial process

It is necessary to simulate at ground level the running conditions of the equipments to test an air conditioning system. More particularly, carrying out these

experiments without running the jet engine requires to be able to simulate the thermodynamic conditions at the high-pressure stages. These conditions fluctuate rapidly and on a wide scale in accordance with the engine rating. Thus, regarding the tests carried out at the CEAT, our test stand will have to satisfy a very strict specification sheet : pressure gradients of 10 bar/s and temperature gradients of 100°C/s.

The technical solution which has been chosen to generate the air flow is represented in the following figure :

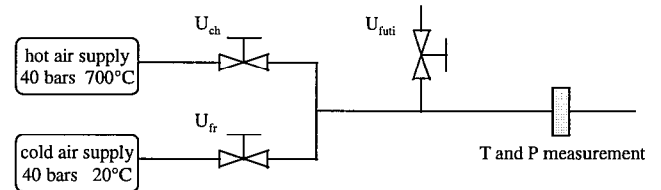


figure 1

The control variables are the 2 valve opening orders  $u_{ch}$  and  $u_{fr}$ , and the output variables are the air flow pressure and temperature (P and T).

The thermodynamic behavior of the system is very different depending on whether the air flow in the P2T2 pipes is subsonic or supersonic. Two differential models have been built up to describe our physical system :

- model in subsonic conditions :

$$\begin{cases} \frac{dP}{dt} = (Au_{ch} + Bu_{fr}) T \sqrt{P_1^2 - P^2} - \alpha QT \\ \frac{dT}{dt} = (Cu_{ch} + Du_{fr}) \frac{T}{P} \sqrt{P_1^2 - P^2} - \beta Q \frac{T}{P} \end{cases}$$

- model in supersonic conditions :

$$\begin{cases} \frac{dP}{dt} = (A'u_{ch} + B'u_{fr}) T - \alpha QT \\ \frac{dT}{dt} = (C'u_{ch} + D'u_{fr}) \frac{T}{P} - \beta Q \frac{T}{P} \end{cases}$$

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The transition from one flow to the other is determined by the difference between the air supplies upper pressure  $P_1$  and the air flow pressure  $P$ : the flow is subsonic (resp. supersonic) when this difference is lower than (resp. higher than) the following limit value :

$$\Delta P_{lim} = P_1 \left( 1 - \sqrt{1 - \frac{10.013}{11.56} C_f^2} \right)$$

Notice that the system is strongly non-linear with additive high coupling effects between the different outputs.

### 3. MPC

#### 3.1 System model

The Generalized Predictive Controller (GPC) has been first introduced by Clarke *et al.* (1987). Later Mohtadi *et al.* (1986), and Shah *et al.* (1987) extended the initial algorithm to the multivariable case in an entirely deterministic framework. More recently, Kinnaert (1989) and Gu *et al.* (1991) proposed a stochastic approach. This paper is based on these two last works, in a more general context.

The multi-input multi-output (MIMO) system is described by the following CARIMA (Controlled Auto-Regressive Integrated Moving Average) model :

$$A(q^{-1})\Delta(q^{-1})y(t) = B(q^{-1})\Delta u(t-1) + C(q^{-1})e(t)$$

$y(t)$ ,  $\Delta u(t-1)$  and  $e(t)$  are the output, the input and the disturbance vectors of respective dimensions  $N \times 1$ ,  $M \times 1$  and  $N \times 1$  ( $M$  being the number of inputs of the system and  $N$  the number of outputs). The  $\{e(t)\}$  sequence is assumed to satisfy :

$$E\{e(t) | F_{t-1}\} = 0 \quad E\{e(t)e(t)^T\} = \sigma_0$$

where  $F_t$  is the  $\sigma$ -algebra generated by the data up to time  $t$ , and  $\sigma_0$  is a positive definite matrix.

$A(q^{-1})$ ,  $B(q^{-1})$  and  $C(q^{-1})$  are polynomial matrices in the unit delay operator  $q^{-1}$  :

$$\begin{aligned} A(q^{-1}) &= A_0 + A_1 q^{-1} + \dots + A_{n_A} q^{-n_A} \\ B(q^{-1}) &= B_0 + B_1 q^{-1} + \dots + B_{n_B} q^{-n_B} \\ C(q^{-1}) &= C_0 + C_1 q^{-1} + \dots + C_{n_C} q^{-n_C} \end{aligned}$$

where the elements  $A_i$ ,  $B_i$  and  $C_i$  are coefficient matrices of respective dimensions  $N \times N$ ,  $N \times M$  and  $N \times N$ .

Moreover  $C(q^{-1})$  is such that  $C(0)=I$  and  $\det C(q^{-1})$  has all its roots strictly outside the unit circle (in the  $q^{-1}$  plane). Finally  $\Delta(q^{-1})$  is the diagonal polynomial matrix :

$$\Delta(q^{-1}) = \text{diag} \left\{ 1 - q^{-1} \right\}_{i=1, \dots, N}$$

#### 3.2 Optimal model predictors

In a classic way we consider  $\Psi(t)$  the filtered output signal :

$$\Psi(t) = P_N(q^{-1}) \{P_D(q^{-1})\}^{-1} y(t)$$

where  $P_N(q^{-1})$  and  $P_D(q^{-1})$  are  $N \times N$  dimensional polynomial matrices used to define the servo behavior of the closed loop system.

Denote the  $j$ -step-ahead optimal predictor of the auxiliary output as :

$$\hat{\Psi}(t+j) = E\{\Psi(t+j) | F_t\}$$

The global predictive model is given by Kinnaert (1989) :

$$\tilde{\Psi}(t) = G \Delta \tilde{u}(t) + \tilde{\Psi}_0(t)$$

with :

$$\begin{cases} \tilde{\Psi}(t) = [\hat{\Psi}(t+1)^T \dots \hat{\Psi}(t+H_p)^T]^T \\ \Delta \tilde{u}(t) = [\Delta u(t)^T \dots \Delta u(t+H_p-1)^T]^T \\ \tilde{\Psi}_0(t) = [\hat{\Psi}_0(t+1 | t)^T \dots \hat{\Psi}_0(t+H_p | t)^T]^T \end{cases}$$

and where  $G$  is the lower-triangular block matrix defined from  $\{[g_0], [g_1], \dots, [g_{H_p-1}]\}$ , the unitary step

response of the system  $P_N \{P_D\}^{-1} \{A\Delta\}^{-1} B$ .

Notice that the elements  $[g_i]$  are  $N \times M$  dimensional matrices.

In order to act upon the frequency spectrum, Gu *et al.* (1991) proposed to take into account an additional controller output weighting through a synthesis filter  $Q_N(Q_D)^{-1}$  :

$$\Phi(t) = Q_N(q^{-1}) \{Q_D(q^{-1})\}^{-1} \Delta u(t)$$

As previously shown, we obtain the following optimal predictor :

$$\tilde{\Phi}(t) = T \Delta \tilde{u}(t) + \tilde{\Phi}_0(t)$$

where  $T$  is also a lower-triangular block matrix and with :

$$\tilde{\Phi}(t) = [\Phi(t)^T \ \Phi(t+1)^T \ \dots \ \Phi(t+H_p-1)^T]^T$$

### 3.3 Criterion function and optimal control law

Consider the following cost function derived from Kinnaert (1989) and Gu *et al.* (1991) :

$$J_1 = E \left\{ \|\bar{\Psi}(t) - \tilde{w}\|_{\Lambda}^2 + \|\bar{\Phi}(t)\|_R^2 \mid F_t \right\}$$

where  $\|x\|_R^2 = x^T R x$  and  $\Lambda$  and  $R$  are weighting diagonal matrices. Denote  $H_p$  the prediction horizon. At each sampling time we form the vector containing the  $H_p$  desired process outputs :

$$\tilde{w}(t) = [w(t+1)^T \ \dots \ w(t+H_p)^T]^T$$

Define :

$$\begin{cases} \bar{\Psi}(t) = [\Psi(t+1)^T \ \dots \ \Psi(t+H_p)^T]^T \\ \bar{\Phi}(t) = [\Phi(t)^T \ \dots \ \Phi(t+H_p-1)^T]^T \end{cases}$$

According to the two optimal predictors presented before, these two vectors satisfy :

$$\begin{cases} E\{\bar{\Psi}(t) \mid F_t\} = \tilde{\Psi}(t) \\ E\{\bar{\Phi}(t) \mid F_t\} = \tilde{\Phi}(t) \end{cases}$$

Thus, the minimization of  $J_1$  is equivalent to the minimization of the following cost function :

$$J_2 = \|\tilde{\Psi}(t) - \tilde{w}(t)\|_{\Lambda}^2 + \|\tilde{\Phi}(t)\|_R^2$$

And the control law is given by :

$$\Delta \tilde{u}(t) = (G^T \Lambda G + T^T R T)^{-1} [G^T \Lambda (\tilde{w} - \tilde{\Psi}_0) + T^T R \tilde{\Phi}_0]$$

The two diagonal matrices  $\Lambda$  and  $R$  are synthesis parameters which allow a relative weighting of tracking errors and controller outputs :

$$\Lambda = \text{diag}_i \{\lambda_i\} \quad R = \text{diag}_i \{r_i\}$$

Moreover, a clever choice of  $\lambda_i$  (or  $r_i$ ) coefficients introduces an additional time weighting : for instance, one could make long-term errors more significant than short-term ones...

### 3.4 Control horizon - Implementation

The preceding control law needs the inversion of a  $(M.H_p) \times (M.H_p)$  dimensional matrix. An additional synthesis parameter can be introduced to reduce the computational cost : the control horizon  $N_u$  (Clarke *et al.* 1987) beyond which every control increments are taken to be zero :

$$\Delta u(t+j) = 0 \quad \text{for } j \geq N_u$$

The solution to our minimization problem becomes a  $N_u \times 1$  dimensional vector :

$$\Delta \tilde{u}_{N_u}(t) = [\Delta u(t)^T \ \dots \ \Delta u(t+N_u-1)^T]^T$$

Thus, the control law is given by :

$$\Delta \tilde{u}(t) = (G_{N_u}^T \Lambda G_{N_u} + T_{N_u}^T R T_{N_u})^{-1} [G_{N_u}^T \Lambda (\tilde{w} - \tilde{\Psi}_0) + T_{N_u}^T R \tilde{\Phi}_0]$$

Now,  $G_{N_u}$  (resp.  $T_{N_u}$ ) is the sub-matrix built from the  $(M.N_u)$  first rows of  $G$  (resp.  $T$ ), and the control law only needs a  $(M.N_u) \times (M.N_u)$  dimensional matrix inversion. Finally, as far as implementation is concerned we will retain the principle of receding horizon (Clarke *et al.* 1987). As minimization computation is repeated at each sampling time, only the  $M$  first lines of the preceding matrix relation are needed to determine the new control increment :

$$\Delta u(t) = L_{MPC} (\tilde{w} - \tilde{\Psi}_0) + M_{MPC} \tilde{\Phi}_0$$

with  $L_{MPC}$  and  $M_{MPC}$   $(M) \times (M.H_p)$  dimensional sub-matrices.

## 4. Multivariable identification

The industrial process has been excited around 11 working points covering all the operating range with uncorrelated PseudoRandom Binary Sequences (PRBS) (Söderström *et al.* 1989) with always the same amplitude. The sampling period was 0.04 s.

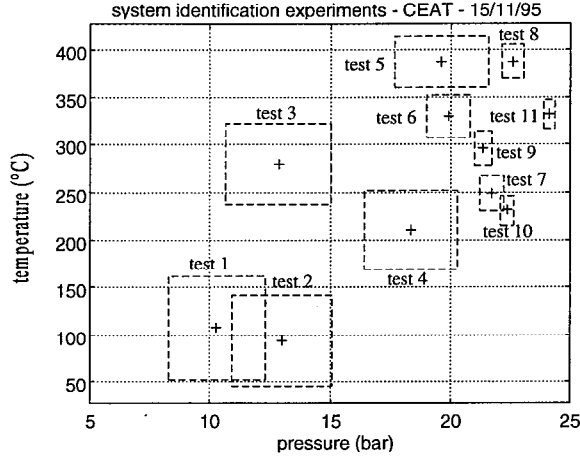


figure 2

'+' mean working point.  
'--' standard deviation.

The standard deviations of the 11 tests clearly reveal the 2 kinds of behavior of the process : the 4 first tests with pressures lower than 18 bars cover the supersonic operating range, while the other tests cover the subsonic operating range.

#### 4.1 Model structure determination

The test stand is controlled through 2 intermediate variables  $u_1$  and  $u_2$  which are combinations of the 2 actuators  $u_{ch}$  and  $u_{fr}$  :

$$u_1 = u_{ch} + u_{fr} \quad u_2 = \frac{u_{ch}}{u_{ch} + u_{fr}}$$

The physical symmetries allow to precise the model structure :

$$\begin{bmatrix} A_{11}(q^{-1}) & A_{12}(q^{-1}) \\ A_{21}(q^{-1}) & A_{22}(q^{-1}) \end{bmatrix} \Delta \begin{pmatrix} P \\ T \end{pmatrix} = \begin{bmatrix} q^{-d_1} B_{11}(q^{-1}) & q^{-d_1} B_{12}(q^{-1}) \\ q^{-d_2} B_{21}(q^{-1}) & q^{-d_2} B_{22}(q^{-1}) \end{bmatrix} \begin{pmatrix} \Delta u_1 \\ \Delta u_2 \end{pmatrix} + C(q^{-1})e(t)$$

$$\text{with : } \begin{cases} \deg(B_{11}) = \deg(B_{12}) = nb_1 \\ \deg(B_{21}) = \deg(B_{22}) = nb_2 \end{cases}$$

$$\text{and the time delays : } \begin{cases} d_{11} = d_{12} = d_1 \\ d_{21} = d_{22} = d_2 \end{cases}$$

Denote  $\varepsilon(t, \theta)$  the linear prediction error at time  $t$  :

$$\varepsilon(t, \theta) = y(t) - \hat{y}(t | \theta)$$

The model structure is obtained minimizing Akaike's Final Prediction Error (FPE) criterion (Ljung 1987) :

$$FPE = \frac{1 + \frac{\dim \theta}{N}}{1 - \frac{\dim \theta}{N}} \times V_N(\hat{\theta})$$

where  $\theta$  is the model parameter vector,  $N$  is the number of data points and  $V_N(\theta)$  is the determinant of the estimated covariance matrix of the innovations. The minimum value has been obtained with :

$$d_1 = 1 \quad d_2 = 5$$

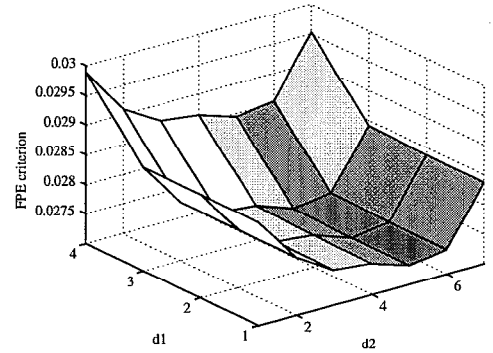


figure 3

and

$$nb_1 = nb_2 = 4$$

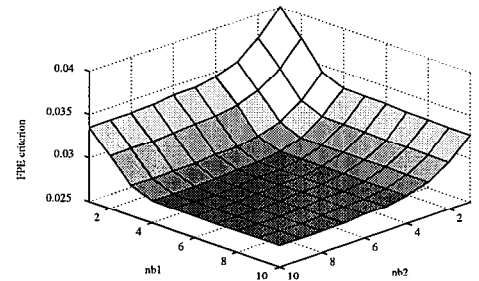


figure 4

Notice the explicit time delay of 5 samples ( $d_2=5$ ) for the control of temperature proved by these identification tests and which has motivated the choice of Long Range Predictive Control (LRPC) methods.

#### 4.2 Mean model - Cross validation

A mean model has been computed minimizing the sum of squared prediction errors with least square methods (Söderström *et al.* 1989). It describes the process behavior over the whole supersonic operating range :

$$\begin{cases} A_{11} = 1 - 0.5359q^{-1} - 0.4031q^{-2} \\ A_{12} = 0.0012q^{-1} - 0.0005q^{-2} \\ A_{21} = -0.4039q^{-1} + 0.3128q^{-2} \\ A_{22} = 1 - 0.5040q^{-1} - 0.4122q^{-2} \end{cases}$$

$$\left\{ \begin{array}{l} q^{-d_1} B_{11} = 0.0762 q^{-1} + 0.1927 q^{-2} \\ \quad \quad \quad + 1.0249 q^{-3} + 1.5084 q^{-4} \\ q^{-d_1} B_{12} = 0.0248 q^{-1} + 0.0411 q^{-2} \\ \quad \quad \quad + 0.2994 q^{-3} + 0.4109 q^{-4} \\ q^{-d_2} B_{21} = 4.1547 q^{-5} + 6.2671 q^{-6} \\ \quad \quad \quad + 7.1529 q^{-7} + 11.6424 q^{-8} \\ q^{-d_2} B_{22} = 9.4131 q^{-5} + 10.0812 q^{-6} \\ \quad \quad \quad + 8.7914 q^{-7} + 17.2238 q^{-8} \end{array} \right.$$

In order to validate this model, we express the correlation rates  $\rho_{PP_s}$  and  $\rho_{TT_s}$  between the measured outputs (P and T) and the simulated ones ( $P_s$  and  $T_s$ ) obtained with the same input PRBS. This work has been done with the first 4 tests which cover the supersonic operating range.

	test 1	test 2	test 3	test 4
$\rho_{PP_s}$	98.26%	98.69%	99.06%	96.70%
$\rho_{TT_s}$	98.51%	96.82%	98.97%	96.60%

Thus, with correlation rates always greater than 96% this mean model has been validated over the whole supersonic operating range.

## 5. PID / MPC comparison

### 5.1 Existing PID regulation

The industrial process is presently controlled with 2 PID reacting respectively on pressure and temperature deviations.

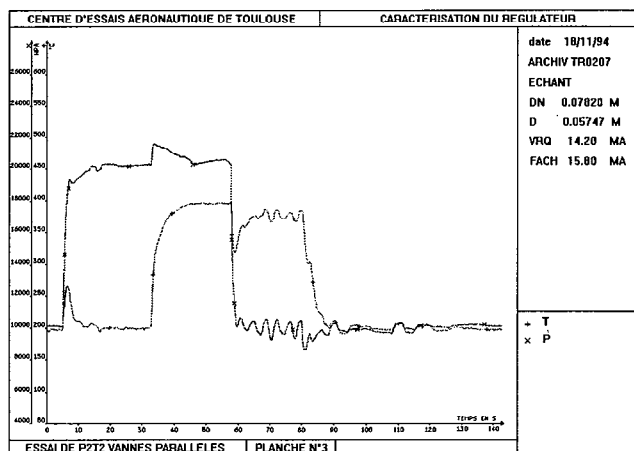


figure 5

The tests presented afterwards have been carried out through a simulation based on the preceding model.

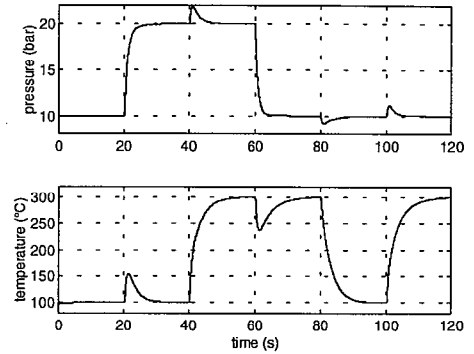


figure 6

Notice the same significant disturbances due to coupling effects between the 2 outputs : the simulation points out deviations of 2 bars for the pressure and 50°C for the temperature.

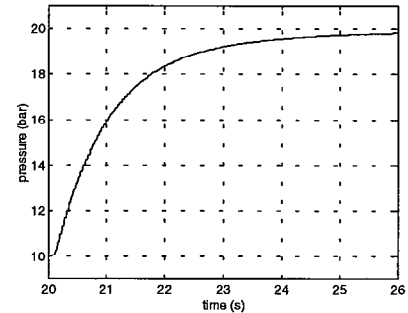


figure 7

With this 10 bar step, the 95% rise time is 3.72 s.

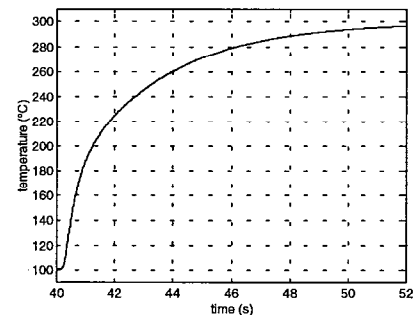


figure 8

In the same way, the 95% rise time with this temperature step of 200°C is 8.40 s.

Thus, the dynamic performances reached through this PID regulation do not satisfy the specification sheet presented before.

## 5.2 MPC regulation

The MPC regulator presented before has therefore been used for this 2-input 2-output system.

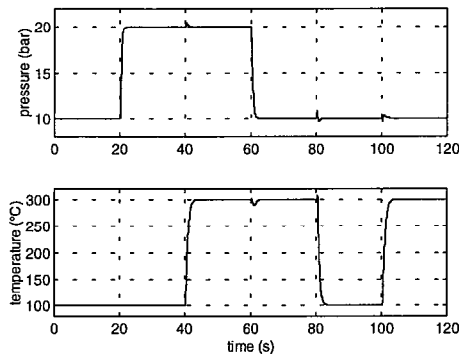


figure 9

In order to compare this MPC regulator with the existing PID, the preceding test has been carried out again.

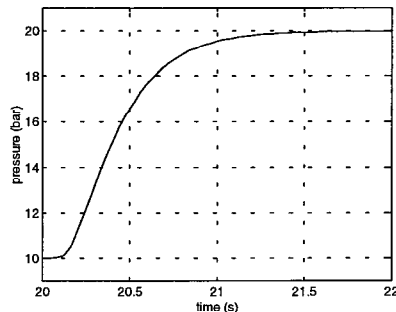


figure 10

The 95% rise time with this 10 bar step is 0.98 s.

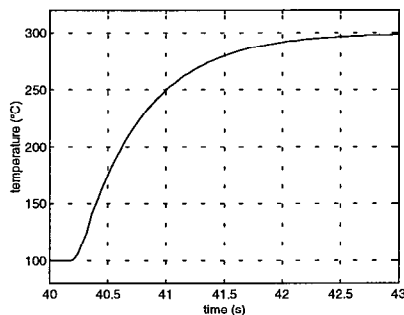


figure 11

And as far as temperature is concerned, the 95% rise time with this 200°C step is 1.88 s.

Thus, the MPC has improved the dynamic performances up to satisfy entirely the specification sheet. Moreover, coupling effects have explicitly been taken into account through optimal predictors and so have been reduced on a large scale : maximum deviations are kept smaller than 0.8 bars for the pressure and 11°C for the temperature.

## 6. Conclusion

In this paper a Multivariable Predictive Controller has been proposed in a stochastic framework for a M-input N-output system. It has been investigated using a simulation study based on an experimental model of an industrial test stand of air conditioning. Comparisons with the existing PID regulation show a great improvement : both step response and coupling effect limitation have been improved.

With a 32 ms calculation time on a PC with 486DX processor (or 8 ms with a Pentium 100 processor), this regulator is able to answer the problems raised by this industrial test stand. Compatible with the industrial regulation hardware, this control algorithm will be soon set up and tested to lead the future air conditioning tests.

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